Medians of Median Analysis

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Q1.

Assuming that the length is repeatedly divisible by all the cuts m, we shall start with m=7

Similarly, we will have the first part in the master theorem to be for finding the true median of the medians.

Of the groups half of them have their median smaller/bigger than the median of the median, . In each group there are 4 elements that are smaller than the pivot. The extreme case would be that the in these groups containing one group with less than seven elements and the one with the true medians in it.

Thus if we look at the elements number smaller/bigger than the true median, the formula will be giving elements smaller/bigger than the median of medians.

Therefore, the worst-case will be a & cut.

Therefore, we have , suppose meets the condition, will satisfy the condition. that there exists constant such that , giving that the algorithm running time . For an active constraint,

Similarly, when b=9, the minor side will be , giving in the worst case, therefore, will require a k meeting the condition that will satisfy the condition, for example, .

When b=11, , b=13, . For a general odd cut m, we will have , similarly we have a active condition on k:

Therefore, the recursive formula will be for .

Q2.1

Following the formula provided in the assignment, we have

When m=7, ignoring the constants in Q1 when n is big, we will have & cut,

When m=9, we will have & cut and .

When m=11, we will have .

When m=13, we will have .

As we put the term into the formula of , we will have:

Q2.2

Using the derived equations in Q2.1, the minimum is dependent on the constant for n. If we compare the constants for , we can see that when m=9, the constant has the smallest value. Therefore, there must be an optimal m for to be smallest.

We can show this conclusion holds in general. That under given n, we can see that the slope function is a tick-shaped curve, and have its minimum at m=9 or m=11, at these two points the function value will be extremely close to each other. The function plot is shown below.

